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## GEOMETRIC STRUCTURES AND DIFFERENTIAL EQUATIONS ON FILTERED MANIFOLDS

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A *filtered manifold*  $(M, F)$  is a differential manifold  $M$  endowed with a filtration  $F = \{F^p\}_{p \in \mathbb{Z}}$  of the tangent bundle  $TM$  of  $M$  satisfying the following conditions:

- (1) Each  $F^p$  is a subbundle of  $TM$  and  $F^p \subset F^{p+1}$ .
- (2)  $F^0 = 0$ , and  $\bigcup F^p = TM$ .
- (3)  $[\underline{F}^p, \underline{F}^q] \subset \underline{F}^{p+q}$ , where  $\underline{F}$  denotes the sheaf of the sections of  $F$ .

Let  $(M, F)$  be a filtered manifold and  $x$  be a point in  $M$ . Denoting by  $F_x$  the fiber of  $F$  over  $x$  and putting  $gr_p F_x = F_x^p / F_x^{p-1}$ , we form a graded vector space

$$gr F_x = \bigoplus gr_p F_x.$$

This vector space has a natural Lie algebra structure induced from the bracket operation of vector fields and satisfies:

$$[gr_p F_x, gr_q F_x] \subset gr_{p+q} F_x.$$

Thus  $gr F_x$  turns out to be a nilpotent graded Lie algebra and may be regarded as a tangent algebra to  $(M, F)$  at  $x$ .

We call *nilpotent geometry and nilpotent analysis* studies of geometric structures and differential equations based on these tangent nilpotent Lie algebras.

The nilpotent geometry has proved to be very fruitful: It gives us, on one hand, unified view points and on the other hand, refined method to study various geometric structures.

A systematic study of differential equations on a filtered manifold  $(M, F)$ , on the basis of *weighted orders* of differential operators associated with  $gr F$ , gives rise to a non-trivial generalization of Cartan-Kähler theorem, a general existence theorem of analytic solutions to system of non-linear analytic partial differential equations possibly with singularities.